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FIXED POINT RESULTS FOR LOCALLY HARDY ROGERS-TYPE CONTRACTIVE MAPPINGS FOR DISLOCATED CONE METRIC SPACE

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Abstract. The purpose of this paper is to introduce some fixed point results for Hardy Rogers-type contractive mappings on a closed ball in ordered complete dislocated cone metric space without continuity. Example has been given to demonstrate the variety of our results. Our result combine, extend and infer several comparable results in the existing literature.

Keywords: fixed point, complete dislocated cone metric space, closed ball.

AMS Subject Classification: 46S40; 47H10; 54H25.

1. Introduction

Let $H : W \longrightarrow W$ be a mapping. A point $w \in W$ is called a fixed point of H if $w = Hw$. In literature, there are many results about the fixed point of mappings that are contractive over the whole space. It is possible that $H : W \longrightarrow W$ is not a contraction but H has a fixed point. Fixed point for such mappings can be obtained by applying necessary and sufficient conditions. It has been shown by Hussain et al. [6], the presence of fixed point for such mappings that fulfill the certain conditions on a closed ball (see also [3, 4, 10-18]). Haung and Zhang [5] have introduced the concept of cone metric space, replacing the set of real numbers by an ordered Banach space. They have proved fixed point theorems of contractive type mappings on cone metric spaces. Lateral, many authors generalized their results. Abbas et al. [1] proved the common fixed point results for non commuting mappings without continuity in cone metric space. After this Altun et al. [2] discussed the fixed point and common fixed point theorems on ordered cone metric spaces. Ilić et al. [7] showed common fixed point for sequences of mappings in cone metric space and generalized some previous results. Further results on cone metric spaces can be seen in [8, 15, 9]. Zhao et al. [19] have established some fixed point results for multivalued mappings in Hilbert spaces. The purpose of this paper to investigate the fixed point results for generalized locally Hardy Rogers-type contraction on closed ball in ordered complete dislocated cone metric space without continuity.

Definition 1.1. [2] *Let E be a real Banach space and P be a subset of E. By θ we denote the zero element of E and by $IntP$ the interior of P. The subset P is called a cone if and only if:*

(i) P is closed , nonempty and $P \neq \{\theta\}$
(ii) $a, b \in$, $a, b \geq 0$, $x, y \in P \Rightarrow ax + by \in P$,
(iii) $x \in P$ and $-x \in P \Rightarrow x = \theta$.
A cone P is called solid if it contains interior points, that is, if $IntP \neq \theta$.

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Given a cone $P \subset E$, we define a partial ordering $\preceq$ with respect to P by $x \preceq y$ if and only if $y - x \in P$. We shall write $x \prec y$ if $x \preceq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in IntP$.

Definition 1.2. [2] *Let X be a nonempty set. Suppose the mapping $d : X \times X \to E$ satisfies:*
(i) If $\theta \prec d(x, y)$ for all $x, y \in X$ with $x \neq y$ and $d(x, y) = \theta$ if $x = y$;
(ii) $d(x, y) = d(y, x)$;
(iii) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.
The pair (X, d) is called a dislocated cone metric space. It is clear that if $d(x, y) = \theta$, then from (i), $x = y$. But if $x = y$, $d(x, y)$ may not be θ. For $x \in X$ and $\varepsilon > 0$, $\overline{B(x, \varepsilon)} = \{y \in X : d(x, y) \preceq \varepsilon\}$ is a closed ball in (X, d).

Definition 1.3. [2] *Let (X, d) be a dislocated cone metric space. Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in E$ with $\theta \ll c$ there is an N such that for all $n > N$, $d(x_n, x) \ll c$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x and x is the limit point of $\{x_n\}$. We denote by this $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$. If for every $c \in E$ with $\theta \ll c$ there is an N such that for all $m, n > N$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X.*

Definition 1.4. *(X, d) is a complete dislocated cone metric space if every Cauchy sequence is convergent.*

2. Main result

Theorem 2.1. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X, d) is complete. Let $x_0 \in \overline{B_d(x_0, r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist α, β, γ, δ, $\eta \geq 0$ with $\alpha + 2\beta + 2\gamma + \delta + \eta < 1$ such that*

$$d(Sx, Sy) \preceq \alpha d(x, y) + \beta d(x, Sy) + \gamma d(y, Sx)) + \delta d(x, Sx) + \eta d(y, Sy) \tag{2.1}$$

for all x, $y \in \overline{B_d(x_0, r)}$ with $y \sqsubseteq x$, and

$$d(x_0, Sx_0) \preceq (1 - \lambda)r \tag{2.2}$$

where $\lambda = \left(\frac{\alpha+\beta+2\gamma+\delta}{1-\beta-\eta}\right)$. If an increasing sequence $\{x_n\}$ in $\overline{B_d(x_0, r)}$ converges to u in $\overline{B_d(x_0, r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0, r)}$.

Proof. If $x_0 = Sx_0$, then the proof is finished. Suppose that $x_0 \neq Sx_0$. Since $x_0 \sqsubseteq Sx_0$ and S is non decreasing with respect to $\sqsubseteq$, we obtain by induction that

$$x_0 \sqsubseteq Sx_0 \sqsubseteq S^2x_0 \sqsubseteq \cdots \sqsubseteq S^n x_0 \sqsubseteq S^{n+1}x_0 \sqsubseteq \cdots .$$

Let $S^n x_0 = x_n$, where $n \in$. Now, using (2.1), we have

$$\begin{aligned} d(Sx_0, Sx_1) &\preceq \alpha d(x_0, x_1) + \beta d(x_0, Sx_1) + \gamma d(x_1, Sx_0) \\ +\delta d(x_0, Sx_0) + \eta d(x_1, Sx_1) &\preceq \alpha d(x_0, x_1) + \beta d(x_0, x_2) + \gamma d(x_1, x_1) \\ +\delta d(x_0, x_1) + \eta d(x_1, x_2) &\preceq \left(\frac{\alpha + \beta + 2\gamma + \delta}{1 - \beta - \eta}\right) d(x_0, x_1) \preceq \lambda(1 - \lambda)r. \end{aligned}$$

Now, we have

$$d(x_0, x_2) \quad \preceq \quad d(x_0, x_1) + d(x_1, x_2) \preceq (1 - \lambda)r + \lambda(1 - \lambda)r \preceq r.$$

This implies that $x_2 \in \overline{B_d(x_0,r)}$. We suppose that $x_3, \cdots, x_j \in \overline{B_d(x_0,r)}$, for some $j \in \mathbb{N}$. Now, using (2.1), we get

$$\begin{aligned}
d(x_j,x_{j+1}) = d(Sx_{j-1},Sx_j) \preceq \alpha d(x_{j-1},x_j) + \beta d(x_{j-1},Sx_j) + \gamma d(x_j,Sx_{j-1}) \\
+\delta d(x_{j-1},Sx_{j-1}) + \eta d(x_j,Sx_j) \preceq \alpha d(x_{j-1},x_j) + \beta d(x_{j-1},x_{j+1}) + \gamma d(x_j,x_{j+1}) \\
+\delta d(x_{j-1},x_j) + \eta d(x_j,x_{j+1}) \preceq \left(\frac{\alpha+\beta+2\gamma+\delta}{1-\beta-\eta}\right) d(x_{j-1},x_j) \\
\preceq \lambda d(x_{j-1},x_j) \preceq \lambda^2 d(x_{j-2},x_{j-1}) \preceq \cdots
\end{aligned}$$

Continuing in this way, we have

$$d(x_j,x_{j+1}) \preceq \lambda^j d(x_0,x_1) \tag{2.3}$$

By using (2.1) and (2.3) we have,

$$\begin{aligned}
d(x_0,x_{j+1}) &\preceq d(x_0,x_1) + d(x_1,x_2) + \cdots + d(x_j,x_{j+1}) \\
&\preceq (1-\lambda)r + \lambda(1-\lambda)r + \cdots + r\lambda^j(1-\lambda) \preceq r
\end{aligned}$$

Thus $x_{j+1} \in \overline{B_d(x_0,r)}$. Hence $x_n \in \overline{B_d(x_0,r)}$ for all $n \in \mathbb{N}$. Now inequality (2.3) can be written as,

$$d(x_n,x_{n+1}) \preceq \lambda^n d(x_0,x_1) \tag{2.4}$$

Using triangle inequality, we get

$$d(x_n,x_{n+i}) \preceq d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + \cdots + d(x_{n+i-1},x_{n+i}),$$

using inequality (2.4), we get

$$d(x_n,x_{n+i}) \preceq \frac{\lambda^n}{1-\lambda} d(x_0,x_1). \tag{2.5}$$

Let $\theta \ll c$. As $c \in IntP$, then

$$N_\varepsilon(\theta) = \{y \in E : \|y\| < \varepsilon\},\ \varepsilon > 0,$$

be a neighborhood of θ such that $c + N_\varepsilon(\theta) \subseteq IntP$. Choose a natural number n_1 such that $\left\|\frac{-\lambda^{n_1}}{1-\lambda} d(x_0,x_1)\right\| < \varepsilon$. Then $\frac{-\lambda^n}{1-\lambda} d(x_0,x_1) \in N_\varepsilon(\theta)$ for all $n \geq n_1$. Hence $c - \frac{\lambda^n}{1-\lambda} d(x_0,x_1) \in c + N_\varepsilon(\theta) \subseteq IntP$. Thus we have

$$\frac{\lambda^n}{1-\lambda} d(x_0,x_1) \ll c \text{ for all } n \geq n_1.$$

Therefore, from (2.5) we get

$$d(x_n,x_{n+i}) \preceq \frac{\lambda^n}{1-\lambda} d(x_0,x_1) \ll c \text{ for all } n \geq n_1$$

Hence we conclude that $\{x_n\}$ is Cauchy sequence in $\overline{B_d(x_0,r)}$. Since $\overline{B_d(x_0,r)}$ is a complete dislocated cone metric space, there exists $u \in \overline{B_d(x_0,r)}$ such that $\{x_n\} \to u$ as $n \to \infty$. As given that $x_n \sqsubseteq u$ for all n, then by using the condition (2.1), we have

$$\begin{aligned}
d(Sx_n,Su) \preceq \alpha d(x_n,u) + \beta d(x_n,Su) + \gamma d(u,Sx_n) \\
+\delta d(x_n,Sx_n) + \eta d(u,Su) \preceq \alpha d(x_n,u) + \beta d(x_n,u) + \beta d(u,Su) \\
+\gamma d(u,Sx_n) + \delta d(x_n,x_{n+1}) + \eta d(u,Su).
\end{aligned}$$

Taking $\lim_{n\to\infty}$ on both sides, we have

$$d(u,Su) \preceq (\beta+\eta) d(u,Su).$$

This implies $(1-\beta-\eta)\,d(u,Su) \preceq \theta$. Therefore, $-d(u,Su) \in P$. As $d(u,Su) \in P$, we have $d(u,Su) = \theta$. Hence $u = Su$. □

Corollary 2.1. *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X,d) is complete. Let $x_0 \in \overline{B_d(x_0,r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist α, γ, δ, $\eta \geq 0$ with $\alpha + 2\gamma + \delta + \eta < 1$ such that*

$$d(Sx,Sy) \preceq \alpha d(x,y) + \gamma d\,(y,Sx) + \delta d(x,Sx) + \eta d(y,Sy)$$

for all x, $y \in X$ with $y \sqsubseteq x$, and

$$d(x_0,Sx_0) \preceq (1-\lambda)r$$

where $\lambda = \left(\frac{\alpha+2\gamma+\delta}{1-\eta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0,r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0,r)}$.

Corollary 2.2. *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X,d) is complete. Let $x_0 \in \overline{B_d(x_0,r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist α, β, δ, $\eta \geq 0$ with $\alpha + 2\beta + \delta + \eta < 1$ such that*

$$d(Sx,Sy) \preceq \alpha d(x,y) + \beta d\,(x,Sy) + \delta d(x,Sx) + \eta d(y,Sy)$$

for all x, $y \in X$ with $y \sqsubseteq x$, and

$$d(x_0,Sx_0) \preceq (1-\lambda)r$$

where $\lambda = \left(\frac{\alpha+\beta+\delta}{1-\beta-\eta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0,r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0,r)}$.

Corollary 2.3. *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X,d) is complete. Let $x_0 \in \overline{B_d(x_0,r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist α, β, γ, $\delta, \geq 0$ with $\alpha + 2\beta + 2\gamma + \delta < 1$ such that*

$$d(Sx,Sy) \preceq \alpha d(x,y) + \beta d\,(x,Sy) + \gamma d\,(y,Sx) + \delta d(x,Sx)$$

for all x, $y \in X$ with $y \sqsubseteq x$, and

$$d(x_0,Sx_0) \preceq (1-\lambda)r$$

where $\lambda = \left(\frac{\alpha+\beta+2\gamma+\delta}{1-\beta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0,r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0,r)}$.

Corollary 2.4. *Let $(X,\sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X,d) is complete. Let $x_0 \in \overline{B_d(x_0,r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$.. Suppose there exist α, β, γ, $\eta, \geq 0$ with $\alpha + 2\beta + 2\gamma + \eta < 1$ such that*

$$d(Sx,Sy) \preceq \alpha d(x,y) + \beta d\,(x,Sy) + \gamma d\,(y,Sx) + \eta d(y,Sy)$$

for all x, $y \in X$ with $y \sqsubseteq x$, and

$$d(x_0,Sx_0) \preceq (1-\lambda)r$$

where $\lambda = \left(\frac{\alpha+\beta+2\gamma}{1-\beta-\eta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0,r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0,r)}$.

Corollary 2.5. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X, d) is complete. Let $x_0 \in \overline{B_d(x_0, r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist $\alpha,\ \delta,\ \eta \geq 0$ with $\alpha + \delta + \eta < 1$ such that*

$$d(Sx, Sy) \preceq \alpha d(x, y) + \delta d(x, Sx) + \eta d(y, Sy)$$

for all $x,\ y \in X$ with $y \sqsubseteq x$, and

$$d(x_0, Sx_0) \preceq (1 - \lambda)r$$

where $\lambda = \left(\frac{\alpha+\delta}{1-\eta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0, r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0, r)}$.

Corollary 2.6. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X, d) is complete. Let $x_0 \in \overline{B_d(x_0, r)}$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist $\gamma,\ \delta,\ \eta \geq 0$ with $2\gamma + \delta + \eta < 1$ such that*

$$d(Sx, Sy) \preceq \gamma d(y, Sx) + \delta d(x, Sx) + \eta d(y, Sy)$$

for all $x,\ y \in X$ with $y \sqsubseteq x$, and

$$d(x_0, Sx_0) \preceq (1 - \lambda)r$$

where $\lambda = \left(\frac{2\gamma+\delta+\eta}{1-\eta}\right)$. If an increasing sequence $\{x_n\}$ converges to u in $\overline{B_d(x_0, r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B_d(x_0, r)}$.

Now we present a new theorem in dislocated cone metric space without closed ball.

Corollary 2.7. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exist a dislocated cone metric d in X such that the dislocated cone metric space (X, d) is complete. Let $x_0 \in X$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist $\alpha,\ \beta,\ \gamma,\ \delta,\ \eta \geq 0$ with $\alpha + 2\beta + 2\gamma + \delta + \eta < 1$ such that*

$$d(Sx, Sy) \preceq \alpha d(x, y) + \beta d(x, Sy) + \gamma d(y, Sx) + \delta d(x, Sx) + \eta d(y, Sy)$$

for all $x,\ y \in X$ with $y \sqsubseteq x$. If an increasing sequence $\{x_n\}$ in X converges to u, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in X.

Metric version of Theorem 4 is given below.

Corollary 2.8. *Let $(X, \sqsubseteq)$ be a partially ordered set and suppose that there exist a cone metric d in X such that the cone metric space (X, d) is complete. Let $x_0 \in X$, $S : X \to X$ be a non decreasing mapping with respect to $\sqsubseteq$ and $x_0 \sqsubseteq Sx_0$. Suppose there exist $\alpha,\ \beta,\ \gamma,\ \delta,\ \eta \geq 0$ with $\alpha + 2\beta + 2\gamma + \delta + \eta < 1$ such that*

$$d(Sx, Sy) \preceq \alpha d(x, y) + y) + \beta d(x, Sy) + \gamma d(y, Sx) + \delta d(x, Sx) + \eta d(y, Sy)$$

for all $x,\ y \in \overline{B(x_0, r)}$ with $y \sqsubseteq x$, and

$$d(x_0, Sx_0) \preceq (1 - \lambda)r$$

where $\lambda = \left(\frac{\alpha+\beta+2\gamma+\delta}{1-\beta-\eta}\right)$. If an increasing sequence $\{x_n\}$ in $\overline{B(x_0, r)}$ converges to u in $\overline{B_d(x_0, r)}$, then $x_n \sqsubseteq u$ for all n. Then S has a fixed point u in $\overline{B(x_0, r)}$.

Example 2.1. *Let* $E =$ *and* $P = \{x \in E : x \succeq 0\}$. *Also, let* $X = [0, \infty)$ *and define a mapping* $d : X \times X \to E$ *by*

$$d(x, y) = x + y, \text{ for all } x, y \in X.$$

Assume that $x \preceq y$, *if and only if* $x \leq y$. *Then the pair* (X, d) *is a dislocated cone metric space. Let* $S : X \to X$ *defined by*

$$Sx = \left\{ \begin{array}{c} \frac{x}{3} \text{ if } x \in [0,1] \\ x+1 \text{ if } x \in (1,\infty). \end{array} \right\}$$

Take $\alpha = \frac{1}{6}$, $\beta = \frac{1}{12}$, $\gamma = \frac{1}{8}$, $\delta = \frac{1}{24}$,and $\eta = \frac{1}{16}$,$x_0 = \frac{1}{4}$, $r = 5$, and $x \sqsubseteq y$, if and only if $x \leq y$, then $\overline{B_d(x_0, r)} = [0, 1]$. We have $\lambda = (\frac{\alpha+\beta+2\gamma+\delta}{1-\beta-\eta}) = \frac{26}{41}$ with

$$(1 - \lambda)r = (1 - \frac{26}{41})5 = \frac{75}{41}$$

and

$$d(x_0, Sx_0) = \frac{1}{4} + \frac{1}{12} = \frac{1}{3} \preceq (1 - \lambda)r.$$

Also if $x, y \in (1, \infty)$. We know that

$$x + y + 2 \geq \frac{1}{6}(x + y) + \frac{1}{12}(x + y + 1) + \frac{1}{8}(y + x + 1) + \frac{1}{24}(2x + 1) + \frac{1}{16}(2y + 1)$$

$$d(Sx, Sy) \geq \alpha d(x, y) + \beta d(x, Sy) + \gamma d(y, Sx) + \delta d(x, Sx) + \eta d(y, Sy)$$

So, the contractive condition does not hold in X. Now, if $x, y \in [0, 1]$, then

$$\frac{x+y}{3} \leq \frac{1}{6}(x+y) + \frac{1}{12}\left(\frac{3x+y}{3}\right) + \frac{1}{8}\left(\frac{3y+x}{3}\right) + \frac{1}{24}(x + \frac{x}{3}) + \frac{1}{16}(y + \frac{y}{3})$$

$$d(Sx, Sy) \leq \alpha d(x, y) + \beta d(x, Sy) + \gamma d(y, Sx) + \delta d(x, Sx) + \eta d(y, Sy).$$

Hence the contractive condition holds only on closed ball rather than whole space. Hence it satisfies all the requirement of Theorem 2.1, so S has a fixed point in closed ball.

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Abdullah Shoaib, for a photograph and biography, see TWMS J.Pure Appl. Math., V.8, N.2, 2017, p.197.

Muhammad Arshad, for a photograph and biography, see TWMS J.Pure Appl. Math., V.8, N.2, 2017, p.197.